5.3 Videos Guide

5.3a

- Shell Method: The volume of a solid obtained by rotating the graph of a function f, $a \leq x \leq b$, $0 \leq a < b$ about the y-axis:
  - $V = \int_a^b 2\pi x f(x)\, dx$
- Note that this is based on the surface area of a cylinder.
  (An analogous formula exists for a function of y.)

Exercises:

- Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.
  $y = x^3,\ \ y = 0,\ \ x = 1,\ \ x = 2$

5.3b

- Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the x-axis.
  $x + y = 4,\ \ x = y^2 - 4y + 4$

- When rotating a region between two curves f and g, $f \geq g$ for $a \leq x \leq b$ (about the y-axis), let $h(x) = f(x) - g(x)$ and we have
  $V = \int_a^b 2\pi x h(x)\, dx$
  (An analogous formula exists for functions of y.)

5.3c

- When the axis of rotation is not a coordinate axis, we can generalize the volume as follows. If the axis of rotation is $x = c$, then let $R = |x - c|$. The above formula becomes
  - $V = \int_a^b 2\pi R h(x)\, dx$

Exercise:

- Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.
  $y = \sqrt{x},\ \ x = 2y;$ about $x = 5$